On quadratic bottom drag, geostrophic turbulence, and oceanic mesoscale eddies

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ABSTRACT  Many investigators have idealized the oceanic mesoscale eddy field with numerical simulations of geostrophic turbulence forced by a horizontally homogeneous, baroclinically unstable mean flow. To date such studies have employed linear bottom Ekman friction (hereafter, linear drag). This paper presents simulations of two-layer baroclinically unstable geostrophic turbulence damped by quadratic bottom drag, which is generally thought to be more realistic. The goals of the paper are 1) to describe the behavior of quadratically damped turbulence as drag strength changes, using previously reported behaviors of linearly damped turbulence as a point of comparison, and 2) to compare the eddy energies, baroclinicities, and horizontal scales in both quadratic and linear drag simulations to observations, and to discuss the constraints these comparisons place on the form and strength of bottom drag in the ocean. In both quadratic and linear drag simulations, large barotropic eddies develop with weak damping, large equivalent barotropic eddies develop with strong damping, and the comparison in 2) above is closest when the nondimensional friction strength parameter is of order one. Typical values of the quadratic drag coefficient \(c_d \sim 0.0025\) and of boundary layer depths \(H_b \sim 50\) m imply that the quadratic friction strength parameter \(c_d L_d / H_b\), where \(L_d\) is the deformation radius, may indeed be of order one in the ocean. Model eddies are realistic over a wider range of friction strengths when drag is quadratic, due to a reduced sensitivity to friction strength in that case. The quadratic parameter is independent of the mean shear, in contrast to the linear parameter. Plots of eddy length scales, computed from satellite altimeter data, versus mean shear and versus rough estimates of the friction strength parameters suggest that both linear and quadratic bottom drag may be active in the ocean. Topographic wave drag contains terms linear in the bottom flow, thus providing some justification for the use of linear bottom drag in models.
1 Introduction

The geostrophic eddy field in the atmosphere and ocean has often been idealized (Salmon 1978, 1980; Haidvogel and Held 1980; Hoyer and Sadourny 1982; Vallis 1983; Hua and Haidvogel 1986; Held and O’Brien 1992; Panetta 1993; Larichev and Held 1995; Held and Larichev 1996; Smith and Vallis 2002; Lapeyre and Held 2003; Arbic and Flierl 2003, 2004a, 2004b; Thompson and Young 2006; Smith 2007; Scott and Arbic 2007; Arbic et al. 2007) with numerical simulations of doubly periodic quasi-geostrophic (QG) turbulence forced by a horizontally homogeneous, baroclinically unstable mean flow. Equilibration in such simulations has thus far been achieved with a linear bottom Ekman friction (hereafter, linear drag). However, drag in bottom boundary layers is usually parameterized as quadratic in the flow (e.g., Gill 1982; Kundu 1990; Holton 1992). Motivated by this, we examine in this paper two-layer baroclinically unstable QG turbulence damped by a bottom quadratic drag. Our two-layer experiments build upon the results of Grianik et al. (2004), who showed that horizontal eddy length scales in forced two-dimensional turbulence damped by a quadratic drag depend on the drag strength but not on the intensity of the forcing (see also Danilov et al. 1994, and Danilov and Gurarie 2000). In contrast, eddy scales in two-dimensional turbulence damped by linear drag depend on the strengths of both drag and forcing (e.g., Smith et al. 2002; Grianik et al. 2004). Here we compare the effects of linear versus quadratic bottom drag in baroclinic turbulence.

The first goal of the paper is to describe the behavior of quadratically damped two-layer turbulence over a wide range of drag strengths. (We refer to turbulence as strongly/moderately/weakly damped when the nondimensional friction strength parameter—which we define in section 3.1—is larger than one/order one/smaller than one, respectively). It
has long been known that QG turbulence damped by weak linear drag is primarily barotropic, and has barotropic length scales much larger than the first baroclinic mode deformation radius $L_d$ (e.g., Salmon 1978, 1980). Arbic and Flierl (2004b; hereafter, AF) explored the strong linear drag limit, in which eddies are equivalent barotropic (much stronger in the upper layer than in the lower layer), and have large length scales of available potential energy. Here we ask whether quadratically damped turbulence also becomes barotropic when drag is weak, and equivalent barotropic when drag is strong. We compare the sensitivities to friction strength exhibited in quadratic versus linear drag simulations, aided by scalings we are able to develop for the strongly damped limit.

The second goal of the paper is to compare the eddy amplitudes, baroclinicities, and horizontal scales in both quadratic and linear drag simulations to those in observations. We use two measures of baroclinicity, the ratios of baroclinic to barotropic kinetic energy, and of upper to lower layer squared velocities. AF showed that eddies in experiments with moderately strong linear drag compare more closely to observations than do eddies damped by either weak or strong linear drag. Here we ask whether moderately strong quadratic drag also yields eddies that compare reasonably well to observations. We discuss the variation of eddy length scales over different regions of the global ocean, and whether such variations can be better explained with the linear drag results, or with the quadratic drag results.

By constraining the types and strengths of drag able to yield simulated eddies that compare well with observations, idealized QG turbulence models can contribute to the discussion of how oceanic eddies are damped. This discussion is an important part of a wider discussion about oceanic dissipation and its consequences (e.g., Munk and Wunsch 1998; Wunsch and Ferrari 2004). Wind forcing inputs about 0.8-1 terrawatt of power into the
geostrophic general circulation (Wunsch 1998; Scott 1999; Huang et al. 2006; Scott and Xu, submitted manuscript). Mesoscale eddies contain much of the kinetic energy of the wind-driven geostrophic circulation. In separate papers under preparation we will discuss energy dissipation of geostrophic flows, and the mechanisms such as bottom boundary layer drag, topographic wave drag, and horizontal eddy viscosity that lie behind energy dissipation, in more detail.

Prompted by the reviewers, we also address here, in more detail than our previous papers, the impact of domain size on our QG turbulence simulations. We have generally performed high-resolution simulations in a relatively small domain, and we discuss the advantages and disadvantages of this choice in section 3.6.

2 The model

We integrate the two-layer equations in the “qgb” model (Flierl 1994). For simplicity, all of the numerical simulations in this paper are done on an f-plane. With quadratic drag, the equations are not invariant to a barotropic translation of the mean flow. We choose to insert the mean flow in the upper layer only, since oceanic mean flows are much stronger at the surface than at the bottom. The f-plane is rotationally invariant, and we take the mean flow to be zonal. With these choices, the governing equations are

\[
\frac{\partial q_1}{\partial t} + \frac{\partial q_1}{\partial x} \frac{\partial \psi_1}{\partial y} + \frac{\partial q_1}{\partial y} \frac{\partial \psi_1}{\partial x} + J(\psi_1, q_1) = \text{ssd},
\]

\[
\frac{\partial q_2}{\partial t} + \frac{\partial q_2}{\partial y} \frac{\partial \psi_2}{\partial x} + J(\psi_2, q_2) = \text{drag} + \text{ssd}.
\]

Subscripts 1 and 2 represent upper and lower layers, respectively. The Jacobian operator

\[ J(A, B) = \frac{\partial A}{\partial x} \frac{\partial B}{\partial y} - \frac{\partial A}{\partial y} \frac{\partial B}{\partial x} \]

Zonal and meridional velocities \((u, v)\), respectively) are defined from the streamfunction \(\psi\), for instance \(\vec{u}_2 = (u_2, v_2) = (-\partial \psi_2 / \partial y, \partial \psi_2 / \partial x)\),
where \( x \) and \( y \) are spatial coordinates in the zonal and meridional directions, respectively. Quantities with overbars represent imposed time-means\(^1\), while quantities without overbars represent fluctuations from the time-mean (eddies). Fluctuation potential vorticity (PV) is given by 

\[
q_1 = \nabla^2 \psi_1 + (\psi_2 - \psi_1)/(1 + \delta)L_d^2 \quad \text{and} \quad q_2 = \nabla^2 \psi_2 + \delta(\psi_1 - \psi_2)/(1 + \delta)L_d^2,
\]

where \( \delta = H_1/H_2 \) is the ratio of layer depths. We perform experiments with both \( \delta = 0.2 \), which is appropriate for the mid-latitude ocean (Flierl 1978; Fu and Flierl 1980), and \( \delta = 1 \), which is more appropriate for the atmosphere and high-latitude ocean. Mean PV gradients are 

\[
\frac{\partial q_1}{\partial y} = \frac{\mathbf{u}}{1 + \delta}L_d^2 \quad \text{and} \quad \frac{\partial q_2}{\partial y} = -\frac{\delta \mathbf{u}}{1 + \delta}L_d^2.
\]

Small scale dissipation (ssd), necessary to remove subgrid scale noise from the model, is accomplished with an exponential cutoff wavenumber filter (see Arbic and Flierl 2003, 2004a for details of our implementation).

For a background flow \( \mathbf{u}_2 \) that is steady on the timescales of boundary layer turbulence, as we expect mesoscale eddy flows to be, the Gill (1982) equations 9.4.6 and 9.5.1 for Ekman pumping \( w_E \) due to quadratic drag are

\[
w_E = \frac{1}{f} \left( \frac{\partial Y_S}{\partial x} - \frac{\partial X_S}{\partial y} \right), \quad X_S = C_d|\mathbf{u}_2|(u_2\cos\alpha - v_2\sin\alpha), \quad Y_S = C_d|\mathbf{u}_2|(u_2\sin\alpha + v_2\cos\alpha). \quad (3)
\]

Here \( f \) is the Coriolis parameter, \( \alpha \) is the angle through which the flow turns inside the boundary layer, and \( C_d = c_d/H_b \), where \( c_d \) is the nondimensional drag coefficient and \( H_b \) is a boundary layer depth. In all of the runs done for this paper save one, we take \( \alpha \) to be zero, as is typically assumed in ocean general circulation models. This is equivalent to setting \textit{drag} in (2) to \(-\hat{z} \cdot (\nabla \times C_d|\mathbf{u}_2|\mathbf{u}_2)\), as was done in Grianik et al. (2004)–note that \( \hat{z} \) is the unit vector in the vertical. We performed one simulation with \( \alpha = 40^\circ \), and found that the results were not greatly different from those in the \( \alpha = 0^\circ \) experiment with equivalent

\(^1\)Since we have set \( \mathbf{u}_2 = 0 \), the mean shear \( \mathbf{u} = \mathbf{u}_1 - \mathbf{u}_2 \) is equal to the mean flow in the upper layer.
other parameters. Note that boundary layers in the actual ocean are very complicated (e.g., Armi 1978; Weatherly and Martin 1978; Lentz and Trowbridge 1991; Garrett et al. 1993; Trowbridge and Lentz 1998; Kurapov et al. 2005; among many). Equation (3), though more complex than linear drag, still represents a gross simplification.

The total energy equation is

\[
\frac{\partial}{\partial t} \int \int \frac{1}{2} \left[ \frac{\delta (\nabla \psi_1)^2}{1 + \delta} + \frac{\delta (\psi_1 - \psi_2)^2}{(1 + \delta)^2 L_d^2} \right] dx dy = \frac{\delta \bar{\Pi}}{(1 + \delta)^2 L_d^2} \int \int \psi_1 \frac{\partial \psi_2}{\partial x} dx dy
\]

\[- \frac{1}{1 + \delta} \int \int (\psi_2 \text{drag}) dx dy + ssd.
\]

(4)

Energies can also be written in terms of the barotropic (BT) and baroclinic (BC) stream-functions \(\psi_{\text{BT}} = (\delta \psi_1 + \psi_2)/(1 + \delta)\) and \(\psi_{\text{BC}} = \sqrt{\delta}(\psi_1 - \psi_2)/(1 + \delta)\). Modal kinetic energies \(KE_{\text{BT}}\) and \(KE_{\text{BC}}\) are given by domain integrals of the densities \((\nabla \psi_{\text{BT}})^2/2\) and \((\nabla \psi_{\text{BC}})^2/2\), respectively, while available potential energy (APE) is the integral of \(\psi_{\text{BC}}^2/2L_d^2\).

Except where noted, the model domain size is \(20\pi L_d\) on a side, and the number of grid-points is \(256^2\). Eddy length scales are defined as reciprocals of the first moments (centroids) of wavenumber spectra. For example, if \(k\) represents (isotropic) wavenumber and \(E_1(k)\) is the spectrum of upper layer kinetic energy, the centroid \(k_{E_1} = \int kE_1(k)dk / \int E_1(k)dk\), and the upper layer length scale is \(1/k_{E_1}\). A domain-filling eddy has a wavelength of \(20\pi L_d\) and a length scale of \(10 L_d\). We refer to the length scales of upper layer kinetic energy, lower layer kinetic energy, APE, \(KE_{\text{BC}}\), and \(KE_{\text{BT}}\) as \(L_1\), \(L_2\), \(L_{\text{APE}}\), \(L_{\text{BC}}\), and \(L_{\text{BT}}\), respectively. Other measures of eddy length scale have been used in the literature (e.g., Stammer 1997; Smith et al. 2002). Our preference for centroids stems from our development of “cascade inequalities” (Arbic et al. 2007), which are written in terms of centroids.
Except where noted, the results we present in this paper are domain- and time-
averages computed after a statistically steady state is reached.

3 Numerical results and scalings

3.1 Identification of nondimensional friction strength parameter

Before presenting the numerical results, we identify the parameter which measures
the strength of quadratic drag. After nondimensionalizing \(x\) and \(y\) by \(L_d\), \(t\) by \(L_d/\bar{u}\), \(\psi_1\)
and \(\psi_2\) by \(\pi L_d\), and \(q_1\) and \(q_2\) by \(\pi/L_d\), the drag term in (2) is preceded by \(C_d L_d\). Thus
\(F_Q = C_d L_d\) is the nondimensional friction strength parameter in quadratic drag simulations.

In linear drag experiments, the friction strength parameter is \(F_L = R_2 L_d/\bar{u}\), where \(R_2\) is
the linear decay rate.\(^2\) We plot results from linear drag experiments against \(F_L\), and from
quadratic experiments against \(F_Q\). By plotting linear and quadratic experiments together
on the same figures, we can compare the sensitivity of quadratically damped turbulence to
\(F_Q\) with the sensitivity of linearly damped turbulence to \(F_L\).

3.2 Sensitivity of eddy statistics to drag strength

This subsection discusses the sensitivity of eddy statistics in the model to drag
strength. Simple arguments lead us to expect that eddy statistics will be less sensitive
to variations in \(F_Q\) in quadratic experiments than they are to variations in \(F_L\) in linear
experiments. Comparison of the quadratic damping term \(-\hat{z} \cdot (\nabla \times C_d \bar{u}_2 | \bar{u}_2|)\) and the linear
damping term \(-R_2 \nabla^2 \psi_2\) in the lower layer PV equation reveals that \(C_d |\bar{u}_2|\) and \(R_2\) play
analogous roles. In the weak drag limit, as \(C_d\) is decreased, \(|\bar{u}_2|\) should increase such that

\(^2\)In our previous papers we used \(1/F_L\), which we called “throughput”, as our nondimensional parameter.

We neglect parameters associated with \(ssd\), which appears to be relatively unimportant for the moderately
damped f-plane case of greatest interest here (Arbic and Flierl 2004a).
the decrease in “effective $R_2$” (i.e., in $C_d |\bar{u}_2|$) is not so great. Likewise, in the strong drag limit, as $C_d$ is increased, $|\bar{u}_2|$ should decrease such that the increase in “effective $R_2$” is not so great. We therefore expect that in both the weakly and strongly damped limits, the model will react to changes in friction strength less dramatically when drag is quadratic than when it is linear.

Figure 1 plots eddy baroclinicities, and horizontal length scales of $APE$, $KE_{BT}$, and $KE_{BC}$, versus friction strength, in $\delta=0.2$ quadratic and linear drag simulations. Figure 1a displays the ratio of upper to lower layer squared velocities $(u_1^2 + v_1^2)/(u_2^2 + v_2^2)$. In both linear and quadratic drag simulations, as friction strength increases in the strongly damped limit, the flow becomes equivalent barotropic (the ratio greatly exceeds unity). As friction strength decreases in the weakly damped limit, the flow becomes barotropic (the ratio approaches unity). As anticipated, in both the strong and weak drag limits, the ratio is less sensitive to variations in friction strength when drag is quadratic than when it is linear. In the strongly damped limit, $KE_{BC}/KE_{BT} \to 1/\delta = 5$ (Fig. 1b; see AF for discussion), but the approach is more gradual when drag is quadratic. Likewise, with weak damping the falloff to values of $KE_{BC}/KE_{BT} \ll 1$ takes place more gradually in quadratic simulations. When quadratic drag is strong and is increased further, $L_{APE}/L_d$ increases (Fig. 1c). However, the increase is less rapid than in the strong linear drag regime. Similarly, as friction is reduced in the weak drag limit, $L_{BT}/L_d$ increases (Fig. 1d), but does so less rapidly in the quadratic case. For both types of drag, $L_{BC}/L_d$ (Fig. 1e) remains near unity for all values of friction strength.

In $\delta=1$ simulations (not shown in Fig. 1), the behavior of $(u_1^2 + v_1^2)/(u_2^2 + v_2^2)$, $KE_{BC}/KE_{BT}$, $L_{APE}$, $L_{BT}$, and $L_{BC}$ is similar to that described above.

Figure 2 plots $L_1/L_d$ in both $\delta=0.2$ and $\delta=1$ simulations, for both linear and quadratic
drag. In the weakly damped limit, $L_1/L_d$ increases with further decreases in friction, due to the large increase of $L_{BT}/L_d$ (Fig. 1d). For both values of $\delta$, $L_1/L_d$ increases more rapidly when drag is linear than when it is quadratic. For both linear and quadratic drag, the increase in $L_1/L_d$ is more rapid when $\delta=1$ than when $\delta=0.2$. This is because the baroclinic and barotropic kinetic energies are weighted equally in the upper layer when $\delta=1$, whereas baroclinic kinetic energy (which has smaller length scales) is weighted heavily in the upper layer when $\delta=0.2$ (see eqn. 11 of AF; Wunsch 1997; Smith and Vallis 2001).

Figures 3a and 3b plot $APE$ and $KE_{BT}$, respectively, in $\delta=0.2$ simulations. The $APE$ increases with changing friction in both the strong and weak drag limits, while $KE_{BT}$ increases with decreasing friction in the weak drag limit. In all cases, the increase is more gradual when drag is quadratic. Energies in $\delta=1$ simulations (not shown) behave similarly.

3.3 Scaling for the strong drag regime

We now discuss our attempts to explain the sensitivities of eddy statistics to friction strength with scaling theories. We have tried but failed to derive a closed scaling (a scaling completely determined by externally imposed parameters) for the weakly damped limit. In the weak quadratic drag case, we could implement a Held and Larichev (1996) style scaling with the eddy length scale taken to be $C_d^{-1}$. Held (1999) suggested that $C_d^{-1}$ would be the eddy scale when drag is quadratic, and this prediction is found to be true in forced-dissipated two-dimensional turbulence (Grianik et al. 2004). However, as seen in Figs. 1 and 2, in two-layer quadratic drag simulations eddy length scales increase much more slowly than $C_d^{-1}$. We therefore choose not to proceed further with this idea. Smith and Vallis (2002) pointed out the difficulties of formulating closed scaling theories for weakly damped f-plane turbulence. The scaling of Larichev and Held (1995), for example, is not closed, since it
takes the eddy length scales that arise in the simulations as empirical inputs. Thompson and Young’s (2006) scaling differs substantially from earlier scalings, but in the end also relies on an empirical numerical result (the exponential dependence of mixing length on $F_L$).

Until closed scalings for weakly damped f-plane turbulence are developed, it will be difficult to explain quantitatively the differing sensitivities to drag strength seen in weakly damped quadratic versus linear drag simulations.

We have managed to obtain a closed scaling for the strong quadratic drag regime. We follow the scaling in AF for the strong linear drag limit. First, we write the rate of energy cascade $\epsilon_c$ as

$$
\epsilon_c \sim \frac{\delta U_1^3}{(1 + \delta)L_{APE}},
$$

where $U_1$ is an upper layer velocity scale. We use $L_{APE}$ as the length scale since it is the scale that varies the most as friction strength changes in the strong drag limit. We have assumed that the upper layer dominates the cascade, since lower layer flow is weak when friction is strong. Next, energy production $\epsilon_p$ is scaled from (4) as

$$
\epsilon_p = \frac{\delta \nabla \psi}{(1 + \delta)^2 L_d} \frac{\partial \psi_2}{\partial x} \sim \frac{\delta \nabla \psi U_1 U_2 L_{APE}}{(1 + \delta)^2 L_d},
$$

where $U_2$ is a lower layer velocity scale. The scalings (5) and (6) for $\epsilon_c$ and $\epsilon_p$ are identical in the strong linear and strong quadratic drag cases, but the relation for energy dissipation $\epsilon_d$ differs for the two types of drag. In the linear drag case, $\epsilon_d$ is scaled from (8) in AF as $\epsilon_d \sim R_2 U_2^2/(1 + \delta)$. In the quadratic case, we replace $R_2$ with $C_d U_2$ to obtain

$$
\epsilon_d \sim \frac{C_d U_2^3}{1 + \delta}.
$$

Our final relation is an adaptation of (14) in AF. The latter was obtained after RMS evaluation of terms in the lower layer PV equation. In the strongly damped limit, the dominant
balance is $\partial q_2/\partial t \sim R_2 \nabla^2 \psi_2$. Assuming that the lower layer timescale is $L_{APE}/U_1$ and that $\psi_2 \ll \psi_1$ so that $q_2 \sim \delta U_1 L_{APE}/(1 + \delta)L_d^2$, we arrive at (14) of AF—i.e.

$(U_1/L_{APE})[\delta U_1 L_{APE}/(1 + \delta)L_d^2] \sim R_2 U_2/L_{APE}$. For the strong quadratic drag scaling, we replace $R_2$ in (14) of AF by $C_d U_2$ to obtain

$$\frac{U_1}{L_{APE}} \frac{\delta U_1 L_{APE}}{(1 + \delta)L_d^2} \sim \frac{C_d U_2^2}{L_{APE}}. \quad (8)$$

By setting $\epsilon_c = \epsilon_p = \epsilon_d$, and using (8), we obtain

$$U_1 \sim \bar{u}, \quad L_{APE} \sim \frac{(1 + \delta)^\frac{3}{2}}{\delta^\frac{1}{2}} L_d F_Q^\frac{1}{2}, \quad APE \sim \frac{\delta \psi_1^2}{2(1 + \delta)^2 L_d^2} \sim \frac{\delta U_1^2 L_{APE}^2}{2(1 + \delta)^2 L_d^2} \sim \frac{\delta^\frac{3}{2}}{2(1 + \delta)^\frac{1}{2}} \bar{u}^2 F_Q^\frac{3}{2}, \quad \epsilon_p \sim \frac{\delta^\frac{5}{2}}{(1 + \delta)^\frac{1}{2}} F_Q^\frac{1}{2}. \quad (9)$$

Figure 4 (right halves of each subplot) shows that our scaling for strong quadratic drag works well qualitatively, but not quantitatively. Consistent with the numerical results, the scaling predicts that $L_{APE}$ (Fig. 4a) and $APE$ (Fig. 4b)$^3$ increase with increasing friction in the strongly damped limit. Upper layer velocities (Fig. 4c) remain nearly constant, as predicted, but at a level somewhat larger than the mean (see AF for a discussion of similar behaviors in the linear drag case). Lower layer velocities (Fig. 4d) and energy production $\epsilon_p$ (PV flux; Fig. 4e) both decrease with increasing friction. As in the strong linear drag scaling, dependencies on $\delta$ in the strong quadratic drag scaling are not always borne out in the numerical experiments, and the quantitative agreement of the numerical results with the scalings is often poor. The quantitative mismatch may in part be due to domain size effects, which will be discussed shortly.

$^3$Note that in the corresponding figure of AF (9b), $APE$ values are mistakenly low by a factor of 2.
In the strong linear drag scaling (15) of AF, the dependencies on $F_L$ are: $U_1 \sim \text{constant}$, $L_{APE} \sim F_L^{\frac{3}{2}}$, $APE \sim F_L^{\frac{5}{2}}$, $U_2 \sim F_L^{-\frac{3}{2}}$, and $\epsilon_p \sim F_L^{-\frac{1}{3}}$. Comparison of these exponents with the exponents in (9) here reveals that (except for $U_1$) all quantities depend less strongly on $F_Q$ in the quadratic scaling than on $F_L$ in the linear scaling. This is in qualitative agreement with the reduced sensitivity to drag strength seen in quadratic versus linear simulations. Figure 5 displays predictions from the scalings, and results from the numerical simulations, for both types of drag, all together on the same plots. Figures 5a and 5b plot $(u_2^2 + v_2^2)/2$ in $\delta=0.2$ and $\delta=1$ simulations, while Figs. 5c and 5d plot $L_{APE}/L_d$ in $\delta=0.2$ and $\delta=1$ simulations. The differences in the slopes of the numerical results for linear versus quadratic drag are matched to some degree by the differences in the slopes of the respective scalings, but the quantitative fits, as noted earlier, are not particularly good.

3.4 Sensitivity of physical flow fields to drag strength

In this subsection we describe the physical flow fields, and how they change as a function of friction strength. We have shown that in the strongly (weakly) damped regime, $L_{APE}$ ($L_{BT}$) is large. Therefore in these limits we expect the potential (kinetic) energy densities, and indeed the flow itself, to be dominated by a small number of large eddies. To gain some understanding of the numbers and sizes of eddies when friction is moderately strong, we extrapolate our strong drag scaling to order one $C_dL_d$ values, which then predicts that $L_{APE}/L_d$ will be near one. Since$^4$ $L_{BC} < L_{APE}$, $L_{BC}/L_d$ is also near one (as previously noted, $L_{BC}/L_d$ remains near one for all values of friction strength). When damping is strong,

$^4$By our definitions, $L_{BC} = \int k^2|\hat{\psi}_{BC}|^2 dk/\int k^3|\hat{\psi}_{BC}|^2 dk$, where $\hat{\psi}_{BC}$ denotes the Fourier transform of $\psi_{BC}$, while $L_{APE} = \int |\hat{\psi}_{BC}|^2dk/\int k|\hat{\psi}_{BC}|^2dk$. By manipulating versions of the Cauchy-Schwartz-Buniakowsky inequality—see for instance entry 196 in Hardy et al. 1952—one can prove that $L_{BC} < L_{APE}$. 13
$L_{BT} \approx L_{BC}$ (see AF for discussion), thus $L_{BT}/L_d \sim 1$. Since $L_{APE}$, $L_{BC}$, and $L_{BT}$ are all relatively small, the moderately damped regime should contain more, and smaller, eddies than do the strongly and weakly damped limits. Since the length scales of potential and kinetic energies are used to construct these arguments for the number of eddies in the domain, it is instructive to examine snapshots of energy density to understand the evolution of the physical flow fields with changing friction strength.

Figure 6 displays snapshots of $APE$ and total $KE$ (barotropic plus baroclinic) density in $\delta=0.2$ simulations, for $F_Q$ values of 100, 1, and 0.01. (Energy densities in the $\delta=1$ simulations behave similarly and are not shown for the sake of brevity). The sensitivity to $F_Q$ resembles the sensitivity to $F_L$ in linear drag simulations described in our earlier papers. In the strongly damped limit, the $APE$ density contains a small number of large structures (Fig. 6a), while the kinetic energy density is comprised of a small number of narrow ribbons (Fig. 6b). In the weakly damped limit, the flow consists of a few widely separated coherent vortices (Figs. 6e, 6f). The order one $F_Q$ regime consists of many horizontally compact, densely packed, irregular weak vortices, with weak tendrils of kinetic energy density (Fig. 6d) wrapped around cores of more diffuse potential energy density (Fig. 6c). As described in AF, the appearance of both kinetic energy density and streamfunction matches that of the ocean much better in the order one friction regime than in the strongly or weakly damped regimes.

3.5 Independence of eddy statistics from mean shear

While $F_L$ depends on the mean shear, $F_Q$ does not. Therefore when drag is quadratic, eddy statistics, once appropriately normalized (e.g., by $\bar{u}^2/2$ for energies, and by $L_d$ for length scales), are independent of the mean shear. Following the suggestion of a reviewer,
we further test this independence with simulations having mean shears that differ from the nominal shear used in the rest of this paper. (As in our earlier papers, we take the nominal shear to have a dimensional value of 1 cm s$^{-1}$). We perform the $\delta=0.2$, $F_Q=0.3$ experiment with values of mean shear $\tau$ four times larger than, four times smaller than, and equal to, the nominal value. Three different random initial conditions (uncorrelated amongst each other) were generated for the large shear, small shear, and nominal shear experiments. We utilize spatially uniform multiplicative factors to adjust the RMS amplitudes of the three initial conditions. The initial condition for the large (small) shear experiment has an amplitude four times larger (smaller) than the amplitude of the initial condition used in the nominal shear experiment. These adjustments make it easier for us to compare the spin-up curves (plots of energy versus time), shown in Fig. 7, for the three experiments. The spin-up curves are very similar, both in their mean levels and in their variability. Furthermore, once appropriately normalized, various eddy statistics--$L_1$, $L_2$, $L_{APE}$, $L_{BC}$, $L_{BT}$, $u_1^2 + v_1^2$, $u_2^2 + v_2^2$, $APE$, $KE_{BC}$, $KE_{BT}$, $\epsilon_p$, and $\epsilon_d$--are within two percent of each other in the three cases. Thus eddy statistics are indeed independent of the mean shear when drag is quadratic.$^5$

3.6 Domain size effects

This subsection examines the importance of the model domain size. For a typical mid-latitude $L_d$ value of 50 km, our domain is dimensionally $\sim 3000$ km on a side, comparable to typical ocean basin sizes. For atmospheric parameters ($L_d \approx 1000$ km), our domain size is about 60000 km on a side, larger than Earth. However, our domain is much smaller than

$^5$At the June 2001 AMS Fluid Dynamics conference, N. Grianik and co-authors showed that the shape of kinetic energy spectra in two-layer turbulence damped by bottom quadratic drag is independent of the mean shear. These (unpublished) results provide more confirmation of our main conclusion in this subsection.
those used in many other QG turbulence studies, which have been more concerned with allowing room for the inverse cascade to proceed than with resolving eddy activity at scales near $L_d$. Strictly speaking, domain size and resolution are independent considerations. As a practical matter, however, computational limitations often lead to a choice between high resolution and large domain size. Given that the ocean and atmosphere are of finite size, and that most of the eddy kinetic energy resides at $L_d$ scales (Stammer 1997), we have chosen to perform high resolution experiments in a relatively small domain.

The choice of resolution affects energy budgets. In our quadratic (linear) drag experiments, the energy dissipation by bottom drag balances energy production to within 1.7 (3) percent or better. We have performed some lower resolution experiments, and in these we find that small-scale damping, now acting at scales near $L_d$, accomplishes about 10 to 20 percent of the energy dissipation. This is consistent with the results in lower resolution runs performed by others (e.g., Thompson and Young 2006). Unless the small-scale damping is observationally (not just numerically) motivated, we agree with Thompson and Young (2006) that it is desirable for model energy dissipations to be performed largely by bottom drag, which is physically and observationally motivated. This is an argument in favor of doing high resolution experiments.

On the other hand, in small domains either the sizes of eddies, or the spacings between them, can in some experiments approach the domain size (see for instance Figs. 6a,b,e,f). In this case the domain size becomes a parameter that affects eddy statistics (e.g., Larichev and Held 1995; Thompson and Young 2006), which is undesirable if one’s main goal is to test scaling theories of eddy properties in homogeneous turbulence.

To measure the impact of domain size on the simulations presented in this paper, we
first inspect wavenumber spectra. Because snapshots were saved infrequently in many of our simulations, it is not possible for us to compute time-averaged spectra here and we instead resort to computing spectra from individual snapshots. Figure 8 presents spectra of $KE_{BT}$ and $APE$ in $\delta=0.2$ experiments with both linear drag (Figs. 8a,b) and quadratic drag (Figs. 8c,d). For both weak linear drag and weak quadratic drag, the barotropic spectra are red (or nearly so) out to the largest scales. For both linear and quadratic drag, the $APE$ spectra of the strongly and weakly damped regimes are red (or nearly so) at the largest scales. The indication is that experiments with extreme values of friction may be affected by domain size.

The spectra of experiments with order one values of either $F_Q$ or $F_L$ are not red (Fig. 8), suggesting that these experiments are not affected by domain size. This is confirmed in Figs. 9a-9d, which show $L_{BT}/L_d$, $L_{APE}/L_d$, $KE_{BC}/KE_{BT}$, and total eddy energy, respectively, in $\delta=0.2$ experiments run with $512^2$ gridpoints in a domain size $(40\pi L_d)$ twice as large as that of the default $256^2$ experiments. The $512^2$ larger domain experiments were performed only for values of $F_L$ and $F_Q$ lying in the range 0.1 to 10. Inspection of Fig. 9 and of other eddy statistics (not shown) indicates that our smaller domain size is perfectly adequate for the order one friction regime of most interest in this paper. The only substantial quantitative differences between the $512^2$ larger domain and $256^2$ nominal domain experiments take place in the linear $F_L=0.1$ run. The statistics of the larger domain $F_L=0.1$ experiment lie even further from the statistics of the $F_Q=0.1$ experiment than do the statistics of the smaller domain $F_L=0.1$ experiment. This strongly suggests that in larger domains the difference between the sensitivities to friction strength in linear versus quadratic drag experiments is likely to be even greater than that documented here in our smaller domain. The major
inferences of section 3.2 regarding the similarities and differences of quadratic and linear drag simulations are strengthened with the inclusion of these larger domain experiments. Note that it is currently impractical for us to perform experiments with either very strong or very weak friction in the $512^2$ larger domain, as such experiments are very expensive computationally even in the $256^2$ smaller domain (see AF for discussion).

4 Comparison of model to observations

In both linear and quadratic drag simulations, eddy amplitudes, baroclinicities, and horizontal scales are near those in observations when friction strength is of order one. In the $\delta=0.2$, $F_Q=0.3$ experiment, for instance, $KE_{BC}/KE_{BT}=1.3$, $(u_1^2 + v_1^2)/(u_2^2 + v_2^2)=26$, $(u_1^2 + v_1^2)/\pi^2=24$, and $L_1/L_d=1.1$. These values are very close to those in the $\delta=0.2$, $F_L=0.4$ linear drag experiment, which AF argued compared reasonably well with oceanic observations. Since both the equivalent barotropic and barotropic cascades are moderated when drag is quadratic, eddy statistics in quadratic drag simulations lie near observations over a wider range of friction strengths than is the case when drag is linear.

Given the sensitivity of the model to drag strength, it is important to discuss the strength of bottom drag in the actual ocean. If one takes $c_d = 0.0025$, a widely used value, $L_d=50$ km, and $H_b=50$ m (Armi 1978), then $F_Q=2.5$. Since order one $F_Q$ values yield model eddies that compare well with observations, the suggestion is that quadratic bottom drag may indeed exert a strong control on ocean eddy statistics.

Because $F_Q$ is independent of the mean shear, our results here suggest that eddy length scales should be independent of shear if bottom drag in the ocean is quadratic. However, because of anticipated changes in both stratification and friction strength with latitude,

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6In the atmosphere, $H_b \sim 1000$ m and $L_d \sim 1000$ km also implies $F_Q \sim 2.5$. 
we expect that the ratio of the length scale $L_{\text{eddy}}$ of surface kinetic energy to the local $L_d$ value should be larger in high latitudes, even as $L_{\text{eddy}}$ itself decreases. Both $F_Q$ and $F_L$ are proportional to $L_d$, which is largely a function of latitude and which decreases by over an order of magnitude from low to high latitudes (Chelton et al. 1998). Therefore friction strength should decrease, and $L_{\text{eddy}}/L_d$ should increase accordingly, as latitude increases. Furthermore, the stratification is different in high latitudes, such that $\delta=1$ is more appropriate than $\delta=0.2$. We checked this by computing global maps of $\delta$ (results not shown). We estimated $\delta$ from the square of the amplitude of the first baroclinic mode at the bottom (Flierl 1987, Table II), and the baroclinic mode structure was found from the World Ocean Atlas climatology (WOA2001; Conkright et al. 2002). As discussed in section 3.2, $L_1/L_d$ increases more rapidly with decreasing friction in $\delta=1$ experiments than in $\delta=0.2$ experiments. (Compare in Fig. 2, for instance, the eddy length scales in moderately damped $\delta=0.2$ experiments versus those in weakly damped $\delta=1$ experiments). The increase of $L_{\text{eddy}}/L_d$ with latitude is seen in altimeter data (e.g., Stammer 1997; see also Scott and Wang 2005, Arbic et al. 2007). AF noted this increase and the possibility that it might be due in part to a decrease in $F_L$ with latitude. However, $F_L$ depends on the mean shear, unlike $F_Q$. In this section we examine with observations the dependence of oceanic eddy length scales on mean shear and on (rough estimates of) $F_L$ and $F_Q$, as a first test of whether eddy length scales are more consistent with linear drag simulations, or with quadratic drag simulations.

Figure 10 plots oceanic values of $L_{\text{eddy}}/L_d$ versus the mean shear. Surface eddy kinetic energy wavenumber spectra are computed from a global satellite altimeter dataset (LeTraon et al. 1998, 2001) over boxes 64 gridpoints on a side\(^7\), where each gridpoint represents $1/3^\circ$

\(^7\)In Arbic et al. (2007) we used boxes 32 gridpoints on a side—the two choices yield very similar results.
of longitude on a Mercator map. In each box the centroid is computed, and $L_{\text{eddy}}$ is taken to be the reciprocal of the centroid. Deformation radii are averaged over the same boxes from values in the Chelton et al. (1998) database. The mean shear is estimated as the magnitude of the thermal wind shear between the surface and 3000 m depth, computed from WOA2001 (Conkright et al. 2002) and averaged over the same boxes. The data are binned by $L_d$ value.

Two general trends can be discerned from Fig. 10. First, as discussed earlier, $L_{\text{eddy}}/L_d$ decreases as $L_d$ increases (so generally $L_{\text{eddy}}/L_d$ is smaller at lower latitudes). Second, in all cases $L_{\text{eddy}}/L_d$ increases with increasing shear, a behavior qualitatively consistent with those in the moderately to weakly damped linear drag simulations. The statistics of the linear relationship between shear and $L_{\text{eddy}}/L_d$ are summarized in Table 1. The correlations vary quite a lot between $L_d$ bins, but were found using a two-sided test with the student t-distribution to be significant at the 5% level for all but two bins: 70 km < $L_d$ < 80 km and 120 km < $L_d$ < 130 km. These two bins had relatively few degrees of freedom, dof, and only span a short range of shear values, c.f. Fig. 10 black line with x’s, and purple line with x’s respectively. The dof of each $L_{\text{eddy}}/L_d$ series were (possibly under) estimated taking into account serial autocorrelation. In particular, we set dof to the minimum of the following three numbers: a) the length of the data series divided by the decorrelation lag inferred from the first zero crossing of the autocorrelation, b) twice the number of changes in sign of the slope less one, and c) the length of the data series.

We caution that the correlations apparent in Fig. 10 may, in some cases, be partially due to confounding of other factors. Recall that $L_{\text{eddy}}/L_d$ is a function of $\delta$ as well as friction strength (the latter depending upon shear for the linear friction case). We found (results not shown) that while $\delta$ and shear were not correlated globally, they did have a significant
correlation in some of the $L_d$ bins used in Fig. 10. The relationships between eddy length scales, stratification, and mean shear in the ocean will be addressed more fully in a future publication.

Table 1 also lists the slopes of least-squares fits computed between the values of $L_{\text{eddy}}/L_d$ and mean shear shown in Fig. 10. The slope is largest in the 20 to 30 km $L_d$ bin, which corresponds to high latitudes. The prominent changes in $L_{\text{eddy}}/L_d$ with shear in high latitudes (consistent with linear drag) may be due to the strong values of mean shear in the Southern Ocean, which imply corresponding decreases in $F_L$ values. To compare our model to the slopes in the data, we construct in Table 2 a list of the (dimensional) slopes expected from four parameter regimes in our linear drag simulations. We compute slopes from both $\delta=0.2$ and $\delta=1$ runs, and for both moderate friction (defined in this exercise as $F_L$ ranging from 0.4 to 0.1) and weak friction (defined in this exercise as $F_L$ ranging from 0.1 to 0.04). The $L_1/L_d$ values in these four regimes (see Fig. 2) always increase monotonically with decreasing $F_L$. The range of $L_1/L_d$ values in these four regimes is listed in Table 2. The assigned range of dimensional shear values is 2-8 cm s$^{-1}$ for the moderate friction cases (hence both assigned shear and $F_L$ vary by a factor of 4), and 2-5 cm s$^{-1}$ for the weak friction case (hence both assigned shear and $F_L$ vary by a factor of 2.5). These shear values are somewhat larger than the nominal 1 cm s$^{-1}$ dimensional shear we’ve assumed in our papers, but are more consistent with the values shown in Fig. 10. Slopes are then computed by differencing the endpoints of the $L_1/L_d$ values and the values of the assigned shear. For instance, the slope in the $\delta=0.2$, $F_L=0.4$ to 0.1 case is $(1.30-1.14)/(8 \text{ cm s}^{-1} - 2 \text{ cm s}^{-1}) = 2.7/(\text{m s}^{-1})$. This slope is of the same order as the slopes in the data (Table 1). The slopes computed in the weak friction $\delta=0.2$ cases, and in both the moderate and weak friction $\delta=1$
cases, are all substantially larger than those seen in the data. These results quantify what is apparent by visual inspection of Figs. 2 and 10—the dependence of eddy length scales on shear in the ocean is not as strong as it is in much of the parameter space explored by the model.

Based on the fact that oceanic eddy length scales are apparently sensitive to shear, but less so than in much of the parameter space covered by our linear drag results, we tentatively suggest that both linear and quadratic drag may be acting in the ocean (with the latter tempering the sensitivity of eddy length scales to shear). Since bottom boundary layers are generally thought to contribute only quadratic drag terms, one might ask whether it is strictly justifiable to use linear bottom drag in models. We believe that topographic wave drag may provide that justification. Topographic wave drag as utilized in the atmospheric community (e.g., Pierrehumbert 1987; Lott and Miller 1997; Scinocca and McFarlane 2000; Garner 2005—see MacCready and Pawluk 2001 and Edwards et al. 2004 for related arguments in an oceanographic context) contains terms both linear and quadratic in the bottom flow. (The quadratic topographic drag is amplified over the value of quadratic drag in flat regions.) Topographic wave drag is a “bottom drag” in the sense that it arises when eddies flow over a rough bottom.\(^8\) Horizontal eddy viscosity is also a linear operator. We will discuss the impact of eddy viscosity on QG turbulence in a separate paper.

In Fig. 11 we present scatter plots of \(L_{\text{eddy}}/L_d\) versus rough estimates of oceanic values of \(F_Q = C_d L_d\) (Fig. 11a), \(F_L = R_2 L_d/\bar{u}\) (Fig. 11b), and \(F_Q + F_L\) (Fig. 11c—we realize that it is nonstandard to add nondimensional parameters). To estimate \(F_Q\), we take

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\(^8\)Some subtleties arise when the vertical deposition of the energy flux is taken into account (i.e., when one considers where in the vertical the waves break), but the base flux is still generated at the bottom.
Values of $L_d$ and of mean shear are computed as described above. A somewhat tighter fit is achieved with an x-axis of $F_Q$ (or with $F_Q + F_L$), suggesting again that quadratic drag may be important in explaining eddy length scales. Note that $L_{edd}/L_d$ is largely defined by latitude, as is revealed in Fig. 11 through use of different colors for different latitude bands. Repeating Fig. 11 with different colors representing instead different values of bottom roughness, where the roughness is inferred from slopes of 1/4° bottom topography derived from the Smith and Sandwell 1997 dataset, revealed no systematic dependence upon roughness–in this latter case different colors appeared randomly distributed about the curve in Fig. 11 (not shown).

The range of $L_{edd}/L_d$ values in quadratic drag simulations (Fig. 2) is not as large as in the data (Figs. 10 and 11). In a forthcoming paper we will discuss the possibility that addition of eddy viscosity improves model agreement with some of the high-latitude behaviors. We speculate that energetic high baroclinic modes, which have smaller horizontal scales, may lie behind the small (less than one) values of $L_{edd}/L_d$ in low latitudes, which are not seen in our two-layer model results.

5 Summary and discussion

Two-layer f-plane baroclinically unstable geostrophic turbulence damped by quadratic bottom drag behaves much like turbulence damped by linear bottom drag, except that the nondimensional friction strength parameter is $F_Q = C_d L_d$ instead of $F_L = R_2 L_d/\overline{u}$. When either linear or quadratic drag is strong, bottom layer velocities are weak, and an increase in $L_{APE}$ ensues as friction is increased. When damping is weak, $KE_{BC}/KE_{BT}$ is much less than one, and $L_{BT}$ increases as friction is decreased. Eddy statistics are less sensitive to the value of drag strength when drag is quadratic than when it is linear. Scalings for the strong
drag limit are consistent with this moderating effect of quadratic drag.

For both linear and quadratic drag, model eddies have realistic amplitudes, baroclinicities, and horizontal scales when friction strength is of order one. Arguments made here suggest that $F_Q$ may indeed be of order one over much of the ocean. Because of the reduced sensitivity described above, eddy statistics stay close to observations over a wider range of friction strengths when drag is quadratic than when it is linear. Because $F_Q$ is independent of the mean shear, eddy statistics (once appropriately normalized) in quadratic drag simulations are independent of the mean shear, in contrast to the statistics in linear drag simulations. Some correlation between $L_{eddy}/L_d$, where $L_{eddy}$ is the length scale of surface kinetic energy computed from satellite altimetry data, and mean shear appears to exist in the ocean, consistent with the results of linear drag experiments. However, the correlations are weaker than those seen in much of the parameter space covered in our linear drag results. All of this suggests (at least to us) that both linear and quadratic bottom drag may be active in the ocean. We argue that the linear bottom drag may be a proxy for topographic wave drag. Plots of $L_{eddy}/L_d$ versus $F_Q$ and $F_L$ indicate a tighter fit for $F_Q$, further suggesting the potential importance of quadratic bottom drag in explaining eddy statistics. The ratio of $L_{eddy}/L_d$ increases with increasing latitude. We argue that this may be due to the combined influence of decreasing values of $F_Q$ and $F_L$ (especially in the Southern Ocean, with its large values of mean shear), and of decreasing stratification, at high latitudes.

Both linear and quadratic drag simulations point to the importance of the “bottomness” of bottom drag. It may seem strange that bottom drag could significantly impact mesoscale eddies, which have surface-intensified velocities. The viewpoint we have taken in our research has been that this surface intensification may result from relatively strong
damping occurring at the bottom. To further emphasize the importance of “bottomness”, Arbic et al. (2007) showed that eddy amplitudes, baroclinicities and horizontal scales in two-layer turbulence damped by Ekman friction of equal strength in the top and bottom layers do not compare well to observations, no matter what the strength of friction is.

Another viewpoint we have taken in our work is that mid-ocean eddies are generated at least in part by local baroclinic instability. If this is true, then the large ratios of eddy to mean kinetic energy in the ocean (e.g., Gill et al. 1974) imply that eddies are strongly non-linear. In turbulence models, nonlinearities drive inverse cascades. Scott and Wang’s (2005) analysis of satellite altimetry data shows that there is indeed an inverse cascade of kinetic energy in the surface geostrophic flow. (Their analysis also implies a source of eddy kinetic energy near to or larger than $L_d$ scales, consistent with local baroclinic instability). Since ocean eddies remain near $L_d$ horizontal scales, the oceanic inverse cascade to large scales cannot be as fully developed as it is in weakly damped turbulence models, which have been widely utilized in eddy parameterization schemes (e.g., Larichev and Held 1995). We think that the weakly (and strongly) damped regimes are useful and interesting asymptotic limits of QG turbulence, but that eddy parameterizations should be based on the order one friction strength regime, which compares better to observations. Eddy parameterizations are also often based on the assumption of a direct (also called forward) cascade of total baroclinic energy. The forward cascade of total baroclinic energy is robust in our model, but the baroclinic kinetic energy undergoes an inverse cascade over a wide range of friction strengths (Scott and Arbic 2007), a fact that also might be of interest in eddy parameterization schemes.

In this as in our earlier papers, we have chosen to perform high resolution experiments in a relatively small domain. We could have borne a lower computational expense had we
chosen instead to perform low resolution experiments in a large domain, with the same number of gridpoints. The latter choice, often taken by QG turbulence researchers, has the advantage that the domain size does not become a parameter affecting the results of experiments in which the eddies become large. Scaling theories such as the one we developed in section 3.3 are thus more appropriately tested in a larger domain. However, our main goal in our QG turbulence research has not been to quantitatively test scaling theories. Rather it has been to describe model behavior as friction strength changes, and to compare the model eddies to eddies in the ocean. There is little doubt that the qualitative behaviors described here—barotropic flow with weak damping, equivalent barotropic flow with strong damping—would hold in domains larger than the ones focused on here. Equally, there can be little doubt that the quantitative values of eddy statistics in our strongly and weakly damped simulations are affected by the small domain size we use. On the other hand, eddy statistics in these limits are not oceanographically realistic, regardless of domain size, and the dependence of eddy statistics on domain size is therefore not our focus here. For eddies in the moderately damped regime, which compares best to observations, our small domain size is perfectly adequate—increasing the domain size in this regime does not affect eddy statistics. The high resolution that we enforce in our small domain size minimizes energy dissipation at small scales by the numerical subgrid scale filter. We view this as a major advantage of our choice to perform high resolution experiments.

This paper presented only f-plane experiments. Turbulence models are made considerably more complex with the addition of planetary beta, especially when the inherent nonzonality of midocean mean flows is taken into account (Spall 2000; Arbic and Flierl 2004a; Smith 2007). A brief exploration of quadratically damped beta-plane turbulence
shows that some of the behaviors in linearly damped beta-plane turbulence survive. For instance, the lattice vortices of Arbic and Flierl (2004a) arise in simulations in which linear drag is replaced by quadratic drag. The energy and isotropy of beta-plane turbulence driven by nonzonal mean flows often lies far from observations. For this reason and for simplicity, our own preference in future research on idealized QG turbulence is to continue to focus mainly on f-plane simulations.

Although adding quadratic drag is likely a step towards realism, our QG turbulence model still contains numerous simplifications not appropriate for the ocean, such as the discretization of continuous stratification to two layers, the horizontal homogeneity of the imposed mean flows, the flat bottom, and the lack of isopycnal outcropping at the surface. We plan to study the effects of quadratic bottom drag and topographic wave drag in more realistic ocean general circulation models, in which some of the deficiencies noted above are rectified. By including tides in realistic models, we can examine the tidal-eddy interactions that take place due to the quadratic nature of bottom drag.

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BKA to think about the impact of quadratic drag on geostrophic turbulence. A number of people at GFDL allowed BKA to run simulations in background on their workstations. The initial simulations and first draft were done in April-June 2001, while BKA was supported by the GFDL/Princeton University Visiting Scientist Program. BKA returned to the work in 2004, while supported by National Science Foundation grant OCE-0327189, submitted the first version of the paper in June 2006, while supported by a Jackson School of Geosciences Development Grant, and completed the work while supported by National Science Foundation grant OCE-0623159 and Naval Research Laboratory grant N000173-06-2-C003.

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http://www.ig.utexas.edu/people/staff/rscott/Resources/rossbyradius.cdf.
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LIST OF FIGURE CAPTIONS

Figure 1: Comparison of baroclinicities and horizontal length scales of modal energies in $\delta = 0.2$ quadratic drag simulations versus $\delta = 0.2$ linear drag simulations. All quantities are plotted against nondimensional friction strength, defined as $F_L = R_2 L_d / \bar{u}$ in the linear drag case and $F_Q = C_d L_d$ in the quadratic drag case.

Figure 2: Horizontal length scales of upper layer kinetic energy as a function of nondimensional friction strength, for $\delta = 0.2$ quadratic (“Quad”) and linear (“Lin”) drag simulations, and for $\delta = 1$ quadratic and linear drag simulations.

Figure 3: Comparison of APE and $KE_BT$ in $\delta = 0.2$ quadratic and linear drag experiments. All quantities plotted against nondimensional friction strength.

Figure 4: Test of the scaling (9) developed for the strongly damped regime, using $\delta = 0.2$ and $\delta = 1$ numerical simulations with bottom quadratic drag. The horizontal axis in all plots is $F_Q$. Extra slanted (horizontal) lines in a,b,d,e (c) are predicted slopes from the scaling. The PV flux $\epsilon_p$ is in units of $\bar{u}^3 / L_d$, while lower and upper layer kinetic energies, as well as APE, are in units of $\bar{u}^2 / 2$. Although the entire set of simulations is plotted, the scaling (9) is meant to apply only to the right half of each plot.

Figure 5: (a) Test of the strong drag scalings for lower layer kinetic energy, normalized by that of the mean flow, in $\delta = 0.2$ numerical simulations having both linear and quadratic drag. The horizontal axis is nondimensional friction strength. Extra slanted lines in the plot are predicted slopes from the scalings, with the steeper (shallower) lines arising from the linear
(quadratic) scaling. (b) As in (a) but for $\delta=1$. (c) Test of the strong drag scalings for $L_{APE}$, in the $\delta=0.2$ linear and quadratic drag simulations. (d) As in (c) but for $\delta=1$. Although the entire set of simulations is plotted, the scaling (9) is meant to apply only to the right half of each plot.

Figure 6: Contours of potential and kinetic energy density, normalized by $u^2/2$, for snapshots of $\delta = 0.2$, $C_dL_d = 100, 1, \text{ and } 0.01$ simulations with bottom quadratic drag. The domain size is $20\pi L_d$ on a side.

Figure 7: Total eddy energy, normalized by $u^2/2$, versus nondimensional time $t\pi/L_d$, for $\delta=0.2$, $F_Q=0.3$ runs in which the shear $\overline{u}$ takes on the nominal value, $1/4$ the nominal value, and $4$ times the nominal value.

Figure 8: Wavenumber spectra of (a) barotropic kinetic energy, for $\delta=0.2$ linear drag simulations with $F_L$ values of $80, 1.25, 0.4, \text{ and } 0.04$, (b) available potential energy, for the same linear drag simulations, (c) barotropic kinetic energy, for $\delta=0.2$ quadratic drag simulations with $F_Q$ values of $100, 3, 0.3, \text{ and } 0.01$, and (d) available potential energy, for the same quadratic drag simulations. Extra horizontal lines are drawn in at the wavenumber corresponding to a length scale of $L_d$.

Figure 9: (a) Barotropic length scale, (b) length scale of potential energy, (c) ratio of baroclinic to barotropic kinetic energies, and (d) total eddy energy, normalized by the kinetic energy of the mean flow, in $\delta=0.2$ nominal domain linear drag simulations ($256^2$ gridpoints; domain $20\pi L_d$ on a side; denoted “Lin”), in $\delta=0.2$ larger domain linear drag simulations ($512^2$ gridpoints; domain $40\pi L_d$ on a side; denoted “Lin, large”), in $\delta=0.2$ nominal domain
quadratic drag simulations ("Quad"), and in $\delta=0.2$ larger domain quadratic drag simulations ("Quad, large"). All quantities plotted against nondimensional friction strength.

Figure 10: Ratios of eddy length scales $L_{\text{eddy}}$, computed from global satellite altimetry data in boxes 21 1/3$^\circ$ on a side, to local $L_d$ values, plotted versus estimated vertical shear between the surface and 3000 m depth. Computations grouped according to bands of $L_d$ values, shown in legend.

Figure 11: Ratios of eddy length scales $L_{\text{eddy}}$, computed from global satellite altimetry data in boxes 21 1/3$^\circ$ on a side, to local $L_d$ values, plotted versus (a) estimated $F_Q$, (b) estimated $F_L$, (c) estimated $F_Q+F_L$. Data from poleward of 50$^\circ$ latitude denoted “high lat”, from between 30$^\circ$ and 50$^\circ$ latitude denoted “mid lat”, and from between 10$^\circ$ and 30$^\circ$ latitude data denoted “low lat”.

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Table 1. Regression of $L_{eddy}/L_d$ on shear. Data is separated into bins of $L_d$ values. Correlation coefficients $r$ between $L_{eddy}/L_d$ and shear within each bin given in second column. Degrees of freedom (dof), and statistical significance (listed as a probability that the correlations occured by chance), are estimated as described in text. Slope of least squares fit line between $L_{eddy}/L_d$ and shear given in last column. Note that there is not enough data in the 100 to 110 km bin to compute eddy statistics.

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<td>0.30811</td>
<td>15</td>
<td>0.22892</td>
<td>1.5595</td>
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<tr>
<td>80 to 90</td>
<td>0.6088</td>
<td>28</td>
<td>0.00035697</td>
<td>3.6356</td>
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<tr>
<td>90 to 100</td>
<td>0.65962</td>
<td>9</td>
<td>0.027232</td>
<td>3.2103</td>
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<tr>
<td>100 to 110</td>
<td>——</td>
<td>——</td>
<td>——</td>
<td>——</td>
</tr>
<tr>
<td>110 to 120</td>
<td>0.83088</td>
<td>7</td>
<td>0.0055274</td>
<td>6.1122</td>
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<tr>
<td>120 to 130</td>
<td>0.27189</td>
<td>11</td>
<td>0.36885</td>
<td>1.1892</td>
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<tr>
<td>130 to 140</td>
<td>0.39978</td>
<td>22</td>
<td>0.05292</td>
<td>2.3593</td>
</tr>
</tbody>
</table>
Table 2. Slopes of $L_{\text{eddy}}/L_d$ versus (dimensional) mean shear, estimated from different parameter regimes of the linearly damped QG turbulence model. The ratio of layer depths, $H_1/H_2$, is denoted by $\delta$. The range of dimensional shears is chosen to start at 2 cm s$^{-1}$ and is varied by factors equal to those in the corresponding ranges of $F_L$ values. Slopes computed as described in text.

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$F_L$ range</th>
<th>$L_1/L_d$ range</th>
<th>Dimensional shears [cm s$^{-1}$]</th>
<th>slope [1/(m/s)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.4 to 0.1</td>
<td>1.14 to 1.30</td>
<td>2 to 8</td>
<td>2.66</td>
</tr>
<tr>
<td>0.2</td>
<td>0.1 to 0.04</td>
<td>1.30 to 2.12</td>
<td>2 to 5</td>
<td>27.33</td>
</tr>
<tr>
<td>1.0</td>
<td>0.4 to 0.1</td>
<td>1.27 to 2.89</td>
<td>2 to 8</td>
<td>27.00</td>
</tr>
<tr>
<td>1.0</td>
<td>0.1 to 0.04</td>
<td>2.89 to 6.09</td>
<td>2 to 5</td>
<td>106.67</td>
</tr>
</tbody>
</table>
Figure 1: Comparison of baroclinicities and horizontal length scales of modal energies in $\delta = 0.2$ quadratic drag simulations versus $\delta = 0.2$ linear drag simulations. All quantities are plotted against nondimensional friction strength, defined as $F_L = R_2L_d/\nu$ in the linear drag case and $F_Q = C_dL_d$ in the quadratic drag case.
Figure 2: Horizontal length scales of upper layer kinetic energy as a function of nondimensional friction strength, for $\delta = 0.2$ quadratic (“Quad”) and linear (“Lin”) drag simulations, and for $\delta = 1$ quadratic and linear drag simulations.
Figure 3: Comparison of $APE$ and $KE_{BT}$ in $\delta = 0.2$ quadratic and linear drag experiments.

All quantities plotted against nondimensional friction strength.
Figure 4: Test of the scaling (9) developed for the strongly damped regime, using \( \delta = 0.2 \) and \( \delta = 1 \) numerical simulations with bottom quadratic drag. The horizontal axis in all plots is \( F_Q \). Extra slanted (horizontal) lines in a,b,d,e (c) are predicted slopes from the scaling. The PV flux \( \epsilon_p \) is in units of \( \overline{u^3}/L_d \), while lower and upper layer kinetic energies, as well as APE, are in units of \( \overline{u^2}/2 \). Although the entire set of simulations is plotted, the scaling (9) is meant to apply only to the right half of each plot.
Figure 5: (a) Test of the strong drag scalings for lower layer kinetic energy, normalized by that of the mean flow, in $\delta=0.2$ numerical simulations having both linear and quadratic drag. The horizontal axis is nondimensional friction strength. Extra slanted lines in the plot are predicted slopes from the scalings, with the steeper (shallower) lines arising from the linear (quadratic) scaling. (b) As in (a) but for $\delta=1$. (c) Test of the strong drag scalings for $L_{APE}$, in the $\delta=0.2$ linear and quadratic drag simulations. (d) As in (c) but for $\delta=1$. Although the entire set of simulations is plotted, the scaling (9) is meant to apply only to the right half of each plot.
Figure 6: Contours of potential and kinetic energy density, normalized by $\frac{u^2}{2}$, for snapshots of $\delta = 0.2$, $C_d L_d = 100$, 1, and 0.01 simulations with bottom quadratic drag. The domain size is $20\pi L_d$ on a side.
Figure 7: Total eddy energy, normalized by $\bar{u}^2/2$, versus nondimensional time $t\bar{u}/L_d$, for $\delta=0.2$, $F_Q=0.3$ runs in which the shear $\bar{u}$ takes on the nominal value, 1/4 the nominal value, and 4 times the nominal value.
Figure 8: Wavenumber spectra of (a) barotropic kinetic energy, for $\delta=0.2$ linear drag simulations with $F_L$ values of 80, 1.25, 0.4, and 0.04, (b) available potential energy, for the same linear drag simulations, (c) barotropic kinetic energy, for $\delta=0.2$ quadratic drag simulations with $F_Q$ values of 100, 3, 0.3, and 0.01, and (d) available potential energy, for the same quadratic drag simulations. Extra horizontal lines are drawn in at the wavenumber corresponding to a length scale of $L_d$. 

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Figure 9: (a) Barotropic length scale, (b) length scale of potential energy, (c) ratio of baroclinic to barotropic kinetic energies, and (d) total eddy energy, normalized by the kinetic energy of the mean flow, in $\delta=0.2$ nominal domain linear drag simulations ($256^2$ gridpoints; domain $20\pi L_d$ on a side; denoted “Lin”), in $\delta=0.2$ larger domain linear drag simulations ($512^2$ gridpoints; domain $40\pi L_d$ on a side; denoted “Lin, large”), in $\delta=0.2$ nominal domain quadratic drag simulations (“Quad”), and in $\delta=0.2$ larger domain quadratic drag simulations (“Quad, large”). All quantities plotted against nondimensional friction strength.
Figure 10: Ratios of eddy length scales $L_{\text{eddy}}$, computed from global satellite altimetry data in boxes $21\,1/3^\circ$ on a side, to local $L_d$ values, plotted versus estimated vertical shear between the surface and 3000 m depth. Computations grouped according to bands of $L_d$ values, shown in legend.
Figure 11: Ratios of eddy length scales $L_{eddy}$, computed from global satellite altimetry data in boxes $21 \frac{1}{3}^\circ$ on a side, to local $L_d$ values, plotted versus (a) estimated $F_Q$, (b) estimated $F_L$, (c) estimated $F_Q+F_L$. Data from poleward of $50^\circ$ latitude denoted “high lat”, from between $30^\circ$ and $50^\circ$ latitude denoted “mid lat”, and from between $10^\circ$ and $30^\circ$ latitude data denoted “low lat”.

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